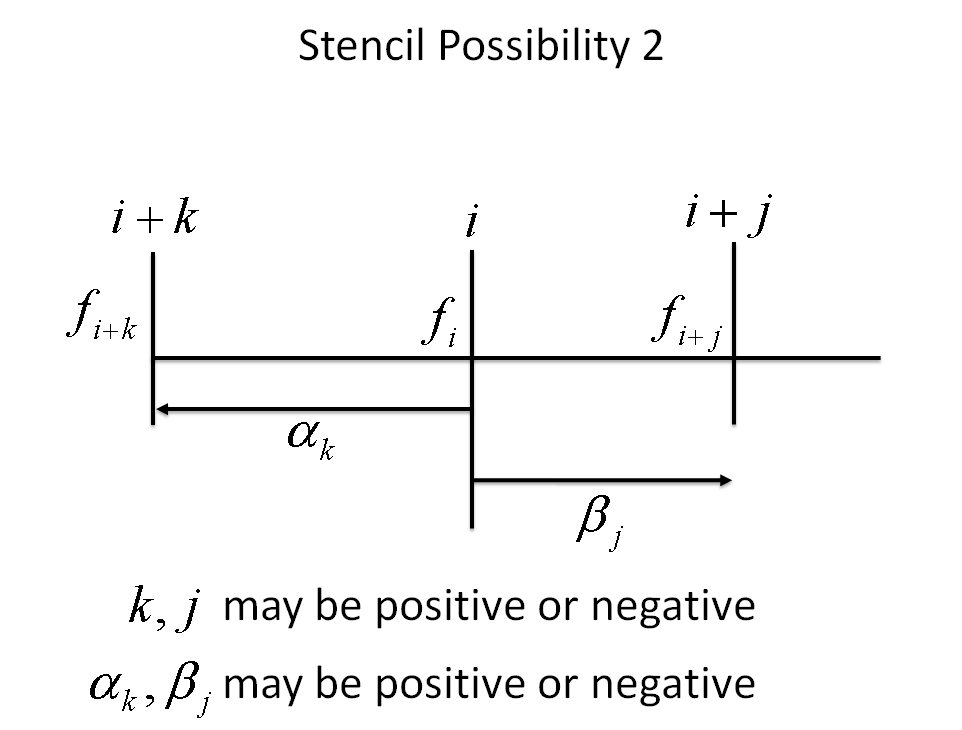
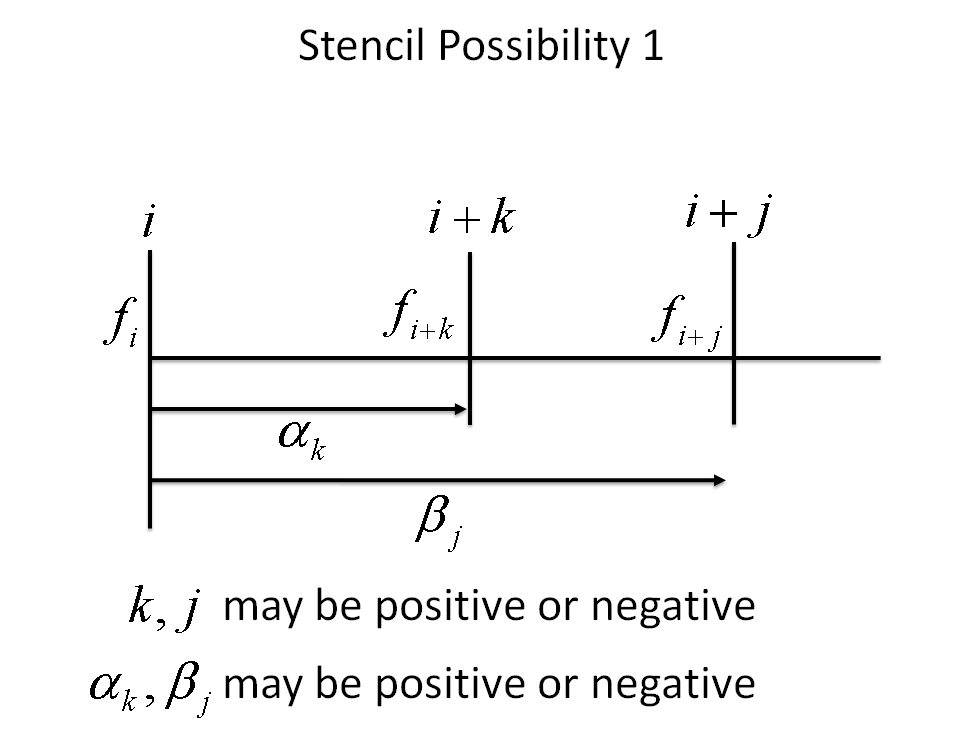
Non-uniform grid stencils

This document contains the derivation of the 1st and 2nd derivative of a function using function values at locations , and with respective indexes. Below are diagrams of the possible stencils that may be made from the results of this document. The following analysis is valid for non-uniform grids.



# Results

These results were obtained by solving the following system of equations

## 1st derivative

## 2nd derivative

# Derivation

A function may be approximated at location from location as

In addition, the function may be approximated at location from location as

Multiplying the first equation by the appropriate coefficient:

Where is some integer, and combining yields

Where is the order, or term, you wish to cancel. Now, to make this analysis more specific, we will aim to focus on 2nd order accurate equations. Therefore, for 1st and 2nd derivatives, will be 2 and 3 respectively. Let's look at these cases separately.

# 1st derivative

For the 1st derivative, we would like to cancel the 2nd order term. i.e.

Which simplifies to

Or

Solving for the derivative at location yields

The order term may also be written as

Doing some algebra this may be written as

This is the 1st derivative of at location using locations and , with order accuracy. is located at and is located at . There is no sign restriction on or . This equation is valid for non-uniform grids.

# 2nd derivative

For the 2nd derivative, we would like to cancel the 3rd order term. i.e.

The problem here is that the 1st derivative still exists, so we must add the equation for the 1st derivative and weight it:

The weight may also be written as

Adding these equations:

Yields

Simplifying the coefficient of the 2nd derivative term

Yields

Solving for the second derivative term, and substituting the previously calculated order of accuracy yields

Collecting terms yields

Let's simplify these terms separately

### First term

Factoring out the alpha yields

### Second term

### Third term

Therefore, our final expression is

Cleaning up we have

# Derivation of order of accuracy for 2nd derivative

This weight of the first equation simplifies to

After dividing by the coefficient, the order of accuracy will drop by a factor of

Therefore, the final order of accuracy will be the sum of the truncation from our 1st and 2nd derivative estimation terms

Simplifying these yields

For and we have

This confirms the simplifying case.